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ARITHMETICAL ERRORS MADE BY HIGH SCHOOL PUPILS.

By J. H. Minnick.

During the school year 1915–16 a review arithmetic test was given to the entering pupils of the William Penn High School of Philadelphia. The questions were prepared by Dr. J. T. Rorer, head of the Department of Mathematics, William Penn High School, and Dr. George W. Flounders, examiner of the Board of Public Education of Philadelphia. An exact copy of the test is given below. The exercises were all printed on one sheet and there was ample space left after each exercise for the solution.

Time: 30 minutes. Name.................... Section......

September 27, 1915.

Review Tests in Arithmetic.

Do all the work on this sheet.

1. Add:

$ 2.34
1.68
2.75
.62
.89
5.48
3.26
.92
.77
7.15
3.50
$

2. Multiply:

97568
7098

3. Divide:

9)270568

4. Divide (without decimals):
1908) 72563 (

5. From fifty-four and five-eighths subtract thirty-four and three-fourths; divide the remainder by two.

6. How many pieces of ribbon, 15½ inches long, may be cut from a 10 yard roll and what is the length of the remnant?

7. Extend the items of the following bill, total, and apply the discount.

Philadelphia, Sept. 27, 1915.

Mr. R. U. Goodpay.

Bought of THE WILLIAM PENN MERCHANDISE COMPANY.

64	yards	Voile	@	$ .62½	per yd.	
48	"	Calico	@	.06¼	" "	
20	"	Lawn	@	.12½	" "	
2	doz.	Fasteners	@	.33⅓	each	
			Less 2% discount,			
			Amount due,			

The instructors of the Department of Mathematics marked the test papers on the basis of "right" or "wrong," without an examination of the specific errors made by the pupils. Later, Dr. Rorer asked me to examine the manuscripts with a view of determining the various types of errors made by the pupils, and the relative frequency of each type. Accordingly, I have carefully examined the manuscripts of a little more than 1,000 pupils. The results* are recorded below.

It should be said that statistics of such an examination can only be approximately correct. The pupil's thoughts are but partially recorded on her paper and in some cases the examiner can only infer the type of error that leads to a given incorrect answer. Furthermore, several errors frequently occur in the same problem, and it is easily possible that some of these have escaped the attention of the examiner. However, we believe that the data given below are fairly accurate.

As the tests were given to incoming pupils who had received little or no training in the William Penn High School, the results must not be taken as an indication of the efficiency of mathematical instruction in that school. They do, however, indicate more or less definitely the store of arithmetical knowledge and habits which the teachers had to use as a basis for their work with these pupils.

In most cases permitting mistakes in number combinations and in "carrying," it was impossible to determine to which type of error an incorrect answer was due. Hence we have not considered these two errors separately, but have classed them as mistakes in addition or multiplication. It is worth noting, however, that in most cases where it was possible to recognize a mistake in "carrying" it was due to inverting the digits of a number. Thus, in $7 \times 8 = 56$ the pupil might have the correct result in mind but record it as 65.

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* Dr. Rorer's report of the test is given in a leaflet published by Dr. Flounders under date of October 25, 1916. A further report was made by Oliver M. Long in *Current Education,* October, 1916.

If an error occurred in all the papers fewer than five times, it has not been included in the tabulation. The only errors possible in Exercise 1 were either in number combinations or in "carrying." These were relatively few, and, as noted above, it was impossible to distinguish between them. Hence the data for Exercise 1 have not been included in this discussion.

Table I gives the data for Exercise 2. Thus, errors in multiplication combination occurred 603 times, which was 78 per cent. of all the errors in the solution of Exercise 2. The errors in multiplying by zero were of two types; namely, using zero as unity and leaving the zero out of consideration in arranging the partial products.

TABLE I.

NUMBER OF TIMES EACH TYPE OF ERROR OCCURRED IN THE SOLUTIONS OF EXERCISE 2.

Type of Error.	Number of Times Each Type Occurred.	Per Cent. of Total Number of Errors.
Multiplication combination	603	78.0
Addition combination	96	12.4
Failure to multiply by one digit of multiplier	23	3.0
Omission of digit in answer or partial product	22	2.8
Multiplying by zero	21	2.7
Failure to multiply part of multiplicand by one digit of multiplier	8	1.0
Total	773	99.9

At first it may seem that failure to multiply by one digit of the multiplier and failure to multiply part of the multiplicand by one digit of the multiplier are the same errors carried to different degrees. This, however, is not the case. In the first the child has overlooked a part of the multiplier, but has used all of the multiplicand; in the second she has neglected a part of the multiplicand while she has used all of the multiplier.

Table II shows that Exercise 3 did not give as much trouble as Exercise 2. The mistake most frequently made in connec-

TABLE II.

NUMBER OF TIMES EACH TYPE OF ERROR OCCURRED IN THE SOLUTIONS OF EXERCISE 3.

Type of Error.	Number of Times Each Type Occurred.	Per Cent. of Total Number of Errors.
Error due to zero	120	61.2
Error in division	44	22.4
Dropped fraction	13	6.6
Error in fraction	10	5.1
Error in subtraction	9	4.5
Total	196	99.8

tion with zero was the omission of a zero in the quotient. That is, the pupils got the same result as if they had divided 9 into 27,568. By an "error in division" is meant an error in finding how many times the 9 is contained in the *next* number of the dividend. Perhaps dropping the fraction 1/9 should not be counted a mistake, as pupils are taught to do so in many cases. The errors in fractions were usually the misplacing of the decimal point or the writing of 1/3 instead of 1/9. A surprisingly large number of pupils used long division, and it was in this connection that the errors were made in subtraction. Another point of interest is the fact that pupils did not notice that 1/9 is a better form for the fraction than .11 1/9.

TABLE III.

NUMBER OF TIMES EACH TYPE OF ERROR OCCURRED IN THE SOLUTIONS OF EXERCISE 4.

Type of Error.	Number of Times Each Type Occurred.	Per Cent. of Total Number of Errors.
Error in division	84	28.6
Error in subtraction	81	27.6
Error in multiplication	57	19.5
Error in handling remainder	23	7.8
Failure to find fraction	18	6.1
Error in copying	16	5.5
Complete confusion	9	3.0
Error in reducing fraction to lowest terms	5	1.7
Total	293	99.8

The data for exercise 4 are given in Table III. The mistakes tabulated under "error in division" are mistakes in determining how many times the divisor was contained in the partial dividends. Of the 84 pupils making such errors, 41 had a digit in their quotients too small. Thirty-one of these 41 pupils noticed that the remainder contained the divisor again. Making this division they obtained two or more digits which belonged in the same place. Thus, in the answer $371^{59}/_{1908}$ the 7 and 1 belong in units place and the answer therefore should be $38^{59}/_{1908}$, which is correct. The remaining ten pupils did not notice that the remainder was larger than the divisor and obtained a result in the form $37^{1967}/_{1908}$.

Of the 23 pupils making errors in handling the remainder, fourteen divided the final remainder by the divisor before expressing it in fractional form. As this remainder was less than the divisor, the resulting digit of the quotient was zero, giving an answer such as $380^{59}/_{1908}$. The remaining 9 pupils failed to divide the last time, resulting in an improper fraction, thus $3^{15323}/_{1908}$. The failure to express the remainder in fractional form perhaps should not be counted an error, as many teachers permit the pupils to express their answer as 38 with a remainder of 59.

The work of 9 pupils was so poorly arranged that it was impossible to follow their thought. In some of these cases it was evident that the pupil had no clear idea of what she was trying to do.

Exercise 5 involves three steps—writing numbers, subtracting, and dividing by 2, and, as Table IV shows, most of the errors can be classified under these three heads. Many of the errors are due to carelessness, such as the failure to divide by 2, multiplying by 2, subtracting the wrong direction and adding instead of subtracting. Twenty-six pupils divided the integral part of the difference by 2, but failed to divide the fraction. This may be due to carelessness, or it may be due to the fact that the pupil had forgotten how to divide fractions by whole numbers. The 5 pupils who divided 2 by the difference, no doubt, had confused the rule for division of fractions and inverted the dividend instead of the divisor.

Dividing the integral part of the difference by 2 gave a remainder of 1. Fifty-six pupils were unable to handle this remainder. The most common mistake was to unite 1 with $^{7}/_{8}$, getting the result $^{17}/_{8}$ instead of the sum of 1 plus $^{7}/_{8}$.

TABLE IV.

NUMBER OF TIMES EACH TYPE OF ERROR OCCURRED IN THE SOLUTIONS OF EXERCISE 5.

Type of Error.	Number of Times Each Type Occurred.		Per Cent. of Total Number of Errors.	
Errors relative to division by 2:				
Failure to divide by 2	137		16.6	
Error in division by 2	67		8.1	
Error in handling remainder	56		6.8	
Failure to divide fraction by 2.	26		3.1	
Multiplied by 2	20		2.4	
Divided 2 by the difference	5		0.6	
Total		311		37.6
Errors relative to subtraction:				
Error in number combinations.	107		12.9	
Subtracted fractions wrong direction	92		11.1	
Error in "borrowing" 1 from 54 and adding it to $\frac{5}{8}$	78		9.4	
Adding instead of subtracting.	59		7.1	
Error in reducing fraction to lowest terms	56		6.8	
Error in reducing mixed number to fraction	14		1.7	
Error in reducing fraction to mixed number	14		1.7	
Total		420		50.7
Error in writing numbers		46		5.6
Error in writing decimals		30		3.6
Complete confusion		15		1.8
Error in copying numbers		5		0.6
Total		827		99.9

Clearly the pupil had confused this case with such problems as 256 divided by 2. Dividing 5 by 2 we get 2 and a remainder of 1. This 1 is 1 ten, and together with 6 units makes 16 units. The pupil learns to do this mechanically, without any thought of the reasons back of the operation. When she divides $9\frac{7}{8}$ by 2, she gets first 4 and a remainder of 1. This 1 and the $\frac{7}{8}$ naturally make $\frac{17}{8}$, just as the 1 and 6 make 16.

Ninety-two pupils subtracted the fractions the wrong direction. This no doubt was suggested by the fact that the fraction of the subtrahend is greater than that of the minuend. Seventy-eight pupils had trouble in "borrowing" 1 from 54 and adding it to $\frac{5}{8}$. Either they did not subtract the 1 from 54, or in adding it to $5/8$ they got $15/8$. Here again the error was due to confusing it with "borrowing" in whole numbers. The errors in the reduction of fractions were usually errors in the fundamental operations.

The errors in decimals usually resulted from an attempt to reduce the original fractions to decimals before performing the subtraction. The solution of 15 pupils indicated that they were completely confused.

Table V gives the data for exercise 6. In all, 1,112 errors

TABLE V.

NUMBER OF TIMES EACH TYPE OF ERROR OCCURRED IN THE SOLUTIONS OF EXERCISE 6.

Type of Error.	Number of Times Each Type Occurred.		Per Cent. of Total Number of Errors.	
Errors in finding remnant:				
Found fraction of piece	463		41.3	
Error in reducing fraction of piece to inches	80		7.1	
Did not know the meaning of remnant	35		3.1	
Did not attempt to find remnant	11		1.0	
Total		589		52.5
Error in method of finding number of pieces:				
Divided length of piece by entire length	35		3.1	
Multiplied length of piece by entire length	26		2.3	
Total		61		5.4
Error in operation:				
Decimals	124		11.1	
Division	50		4.5	
Subtraction	34		3.0	
Multiplication	32		2.9	
Total		240		21.5
Use of denominate numbers:				
Error in reducing from one denomination to another	83		7.4	
Divided without reducing to same denomination	35		3.1	
Total		118		10.5
Complete confusion		114		10.2
Total		1,122		100.1

are recorded in this table. Slightly more than 52 per cent. of these errors were made in finding the remnant. This includes only mistakes in the method of finding the remnant and in the interpretation of the meaning of the word remnant; it does not include mistakes in computation and decimals. Four hundred and sixty-three pupils found the fractional part of a piece left and called it the remnant. Eighty pupils who recognized the real meaning of this fraction made errors in reducing it to inches. Thirty-five pupils did not know the meaning of the word "remnant"; some called the entire piece of ribbon the remnant, while others took the word to mean one of the pieces 15½ inches in length.

The next largest number of errors was made in the use of decimals. Seventy of these were miscellaneous errors in placing the decimal point, 6 were made in reducing common fractions to decimals and vice versa, and the remaining 48 were made in finding the remnant. The pupil divided 360 by 15.5, getting 23 with a remainder of 35 instead of 3.5. The cause of the error is clear. The pupil has learned in a mechanical way to write the fraction $^{35}/_{155}$ omitting the decimal from both numerator and denominator. Hence in this case she very naturally omitted the decimal from the remainder.

The most common mistake in the use of denominate numbers was the confusion of the number of inches and feet in a yard. Many pupils used a yard as equal to 12 inches, and in a few cases they used it as equal to 3 inches.

The work of 114 pupils indicated that they had no idea as to the method of solving the problem. It is probable that in many of these cases the pupil did not visualize the act of cutting the roll of ribbon into pieces 15½ inches long, or if she did, she did not see the relation of the arithmetical operations to this act.

In tabulating the data for exercise 7 (see Table VI), it has been our purpose to show where the errors have occurred, rather than the specific kind of arithmetical errors found in the papers. By far the largest number of errors was made in extending the items of the bill. These were for the most part mistakes in number combinations or in the use of decimals. Fifty-five pupils showed that they did not know the meaning of the word *discount*. Dividing the total by 2, subtracting 2 or .02 from the

TABLE VI.

Number of Times Each Type of Error Occurred in the Solutions of Exercise 7.

Type of Error.	Number of Times Each Type Occurred.		Per Cent. of Total Number of Errors.	
Error in Extensions:				
64 × .62½	138		18.1	
48 × .06¼	123		16.1	
2 × .33⅓	117		15.4	
20 × .12½	70		9.2	
Total		448		58.8
Error in Discount:				
Computation	147		19.3	
Did not know meaning of discount	55		7.2	
Failed to find discount	37		4.9	
Failed to subtract discount	26		3.4	
Total		265		34.8
Error in adding to final total		27		3.5
Error in subtracting discount		22		2.9
Total		762		100.0

total, subtracting half the total from the whole total, and adding the discount instead of subtracting it, are the more common errors resulting from the misconception of discount. In some cases the pupil found the discount and called it the amount due. It is also possible that the 37 pupils who failed to find the discount and the 26 who did not subtract it from the total did not understand the meaning of the word *discount.* Many pupils who had correct answers did not enter the amounts correctly. However, the teachers who originally marked the papers did not take these errors into consideration, and for that reason we have not tabulated them.

It should be emphasized that general conclusions concerning the relative frequency with which errors occur or their relative values cannot be drawn from the results of such a test as this. We do not know that this test gives equal opportunity for all arithmetical errors to occur. However, it is clear that any teacher may easily devise a set of questions which will yield valuable knowledge concerning her classes and give her a fairly definite idea of the arithmetical background which the children have and which she must use as a basis for her work. Also we can draw certain conclusions relative to the teaching of arithmetic. Pupils often have difficulty with those operations which should be reduced to habit. If an operation is to be used frequently, economy of time and accuracy demand that it shall become habitual. This requires not only drill at the time the habit is formed, but also frequent applications in order to keep the habit fixed. Errors in number combinations are due to such incorrect or incomplete habit formation. This is especially true when zero is involved. Most teachers assume that pupils know the combinations of zero with other numbers. This assumption is not true. Even college students frequently have trouble in such cases. It is just as necessary to drill on $0 \times 8 = 0$ as on $9 \times 8 = 72$. If, however, an operation has applications in slightly different forms, an intelligent understanding of it may prevent errors when it is applied in a new form. Thus, in Exercise 5 the pupils "borrowed" 1 from 54 and combined it with $\frac{5}{8}$, getting $\frac{15}{8}$. In dealing with integral numbers they had learned in a mechanical way to "borrow" 1 from 6 in 267 and combine it with 7 so as to get 17. If the pupils had had an intelligent understanding of this operation, if they had known that they were reducing 1 ten to units and adding it to 7 units, and if they had understood that numbers must always be reduced to the same denomination before they can be added, it is possible that such an error would not have occurred so frequently. Errors in application of arithmetic are frequently due to the child's inability to read the problem. This is the case with exercises 6 and 7. Some pupils did not understand the meaning of *remnant* and of *discount*. This does not mean that these words should not be used, but before beginning her solution the pupil should be sure that she understands their meaning. Successful application of arithmetic requires an analysis of the situation to which the application is to be made. If in such problems as Exercise 6, pupils were taught first to see the various operations involved in the physical act described and then to identify the arithmetical operations with the corresponding physical operation, fewer pupils would be confused by such problems.

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